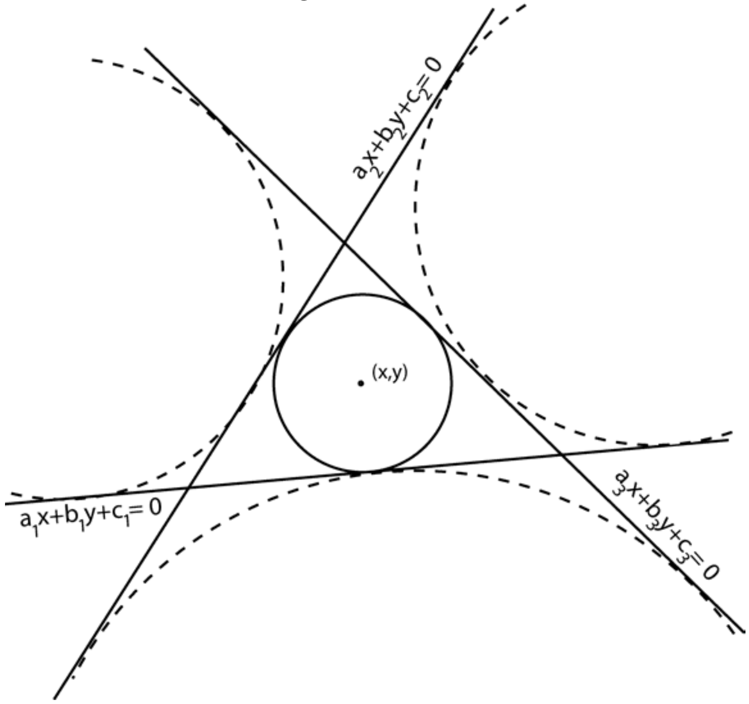
Parameters of a Circle Tangent to Three Lines

Given a line having the equation $a_1x + b_1y + c_1 = 0$; a line having the equation $a_2x + b_2y + c_2 = 0$; and a line having the equation $a_3x + b_3y + c_3 = 0$, it is required to find the circle tangential to these lines, centred at (x, y) with radius *r*. It will be noted that in general there are four solutions.



The line parallel to a line $ax + by + c = 0$ at a distance *d* is given by,

$$ax + by + c - d\sqrt{(a^2 + b^{2)}} = 0$$

If the line equation is normalized, this reduces to,

$$ax + by + c - d = 0$$

The necessary equations for tangentiality are then,

$$a_1x + b_1y + c_1 - rt_1 = 0 \quad \ldots\text{(i)}$$
$$a_2x + b_2y + c_2 - rt_2 = 0 \quad \ldots\text{(ii)}$$
$$a_3x + b_3y + c_3 - rt_3 = 0 \quad \ldots\text{(iii)}$$

where $t_n = \pm 1$

eliminating *y* from (i) and (ii) and from (ii) and (iii) gives,

$$x = \frac{b_2c_1 - b_2rt_1 - b_1c_2 + b_1rt_2}{a_2b_1 - a_1b_2}$$
$$x = \frac{b_3c_2 - b_3rt_2 - b_2c_3 + b_2rt_3}{a_3b_2 - a_2b_3}$$

extracting the constants,

$$u = a_2 b_1 - a_1 b_2$$
$$v = a_3 b_2 - a_2 b_3$$

then,

$$x = \frac{b_2 c_1 - b_2 r t_1 - b_1 c_2 + b_1 r t_2}{u} \qquad \ldots\text{(iv)}$$

$$x = \frac{b_3 c_2 - b_3 r t_2 - b_2 c_3 + b_2 r t_3}{v} \qquad \ldots\text{(v)}$$

equating (iv) and (v) gives,

$$r = \frac{u(b_3 c_2 - b_2 c_3) - v(b_2 c_1 - b_1 c_2)}{v(b_1 t_2 - b_2 t_1) - u(b_2 t_3 - b_3 t_2)} \qquad \ldots\text{(vi)}$$

substitute in (iv) to find x when $u \neq 0$, i.e. lines 1 and 2 not parallel, otherwise in (v)
Reverse the signs of t_n to find all tangential circles.
If the second line is vertical ($b_2 = 0$) the divisor in (vi) is always zero. In this case, exchange lines.